

Name: _____

Signature: _____

TA's Name: _____

Math 1A, Sample midterm

Discussion section: _____

Instructions: *Show your work. Unjustified answers will not receive credit.*

1. (a) State carefully: the Mean Value Theorem.

(b) A particle moves in a straight line with acceleration at time t given by $a(t) = 2t \text{ m/sec}^2$. It has initial velocity $v(0) = 5 \text{ m/sec}$. What is its net change of position between $t = 0$ and $t = 3$?

2. Find the equation of the line tangent to the curve $2(x^2 + y^2)^2 = 25(x^2 - y^2)$ at the point $(3, 1)$.

3. (a) Let $h(x) = f(g(x))$, where f, f', g , and g' are differentiable everywhere. Find $h''(0)$, given that $f'(2) = 5$, $f''(2) = 4$, $g(0) = 2$, $g'(0) = 3$, and $g''(0) = 2$.

(b) Use differentials to estimate the amount of paint needed to apply a coat of paint 0.05 cm thick to a hemispherical dome with diameter 50 cm.

4. A ladder 10 ft. long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a speed of 2 ft/s, how fast is the angle between the top of the ladder and the wall changing when that angle is $\pi/4$ radians.

5. Compute (a) $\lim_{x \rightarrow \infty} (xe^{1/x} - x)$.

(b) $\frac{d}{dt}(t^{\sin(t)})$

(c) $\lim_{x \rightarrow 0^+} \frac{2\sqrt{x}}{\arcsin(x)}$.

6. Let $h(x) = 3x^5 - 5x^3 + 3$. (a) Find the intervals of increase and decrease.

(b) Find the local maxima and minima.

(c) Find the intervals of concavity and the inflection points.

(d) Sketch the graph of h .

7. A cylindrical can without a top is made to contain V cm³ of liquid. Find the dimensions that will minimize the amount of metal required to make the can.

8. A sample of tritium-3 decayed to 94.5 percent of its original amount after one year.

(a) What is the half-life of tritium-3?

(b) How long would it take the sample to decay to 20 percent of the original amount?